Soft \((1,2)\)-Omega Separation Axioms and Weak Soft \((1,2)\)-Omega Separation Axioms in Soft Bitopological Spaces

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Received in: 24/April/2018, Accepted in: 6/June/2018

Abstract

In the present paper we introduce and study new classes of soft separation axioms in soft bitopological spaces, namely, soft \((1,2)\)-omega separation axioms and weak soft \((1,2)\)-omega separation axioms by using the concept of soft \((1,2)\)-omega open sets. The equivalent definitions and basic properties of these types of soft separation axioms also have been studied.

Keywords: Soft \((1,2)\)-\(\omega\)-open sets, soft \((1,2)\)-\(\omega\)-\(\tilde{T}_i\)-spaces, soft \((1,2)\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_i\)-spaces, soft \((1,2)\)-\(\text{pre-}\omega\)-\(\tilde{T}_i\)-spaces, soft \((1,2)\)-\(b\)-\(\omega\)-\(\tilde{T}_i\)-spaces, and soft \((1,2)\)-\(\beta\)-\(\omega\)-\(\tilde{T}_i\)-spaces, for \(i = 0, \frac{1}{2}, 1, 2\).
Introduction

Soft set theory was firstly introduced by Molodtsov [1] in 1999 as a new mathematical tool for dealing with uncertainty while modeling problems in computer science, economics, engineering physics, medical sciences, and social sciences. In 2011 Shabir and Naz [2] introduced and studied the concept of soft topological spaces. In 2014 Senel and Çagman [3] investigated the notion of soft bitopological spaces over an initial universe set with a fixed set of parameters. In 2018 Mahmood and Abdul-Hady [4] introduced and studied new types of soft sets in soft bitopological spaces called soft (1,2)*-omega open sets and weak forms of soft (1,2)*-omega open sets such as soft (1,2)*-alpha-open sets, soft (1,2)*-open-open sets, soft (1,2)*-beta-open sets and soft (1,2)*-open sets. The main purpose of this paper is to introduce and study new types of soft separation axioms in soft bitopological spaces called soft (1,2)*-omega separation axioms and weak soft (1,2)*-omega separation axioms by using the notion of soft (1,2)*-omega open sets such as soft (1,2)*-alpha-open sets, soft (1,2)*-omega-open sets, soft (1,2)*-pre-open sets, space spaces, soft (1,2)*-pre-open sets, soft (1,2)*-beta-open sets, soft (1,2)*-open sets, and soft (1,2)*-beta-open sets , for i = 0, 1, 2. Moreover we study the fundamental properties and equivalent definitions of these types of soft separation axioms.

1. Preliminaries:

Throughout this paper U is an initial universe set, P(U) is the power set of U, P is the set of parameters and \( C \subseteq P \).

**Definition (1.1)** [1]: A soft set over U is a pair \((H, C)\), where H is a function defined by \( H: C \rightarrow P(U) \) and \( C \) is a non-empty subset of \( P \).

**Definition (1.2)** [5]: A soft set \((H, C)\) over U is called a soft point if there is exactly one \( e \in C \) such that \( H(e) = \{u\} \) for some \( u \in U \) and \( H(e') = \emptyset \), \( \forall e' \in C \setminus \{e\} \) and is denoted by \( \tilde{u} = (e, \{u\}) \).

**Definition (1.3)** [5]: A soft point \( \tilde{u} = (e, \{u\}) \) is called belongs to a soft set \((H, C)\) if \( e \in C \) and \( u \in H(e) \), and is denoted by \( \tilde{u} \in (H, C) \).

**Definition (1.4)** [5]: A soft set \((H, C)\) over U is called countable (finite) if the set \( H(e) \) is countable (finite) \( \forall e \in C \).

**Definition (1.5)** [6]: A soft set \((H, C)\) over U is called a null soft set with respect to \( C \) if for each \( e \in C \), \( H(e) = \emptyset \), and is denoted by \( \tilde{0}_C \). If \( C = P \), then \((H, C)\) is called a null soft set and is denoted by \( \tilde{0} \).

**Definition (1.6)** [6]: A soft set \((H, C)\) over U is called an absolute soft set with respect to \( C \) if for each \( e \in C \), \( H(e) = U \), and is denoted by \( \tilde{U}_C \). If \( C = P \), then \((H, C)\) is called an absolute soft set and is denoted by \( \tilde{U} \).

**Definition (1.7)** [6]: Let \((H_1, C_1)\) and \((H_2, C_2)\) be soft sets over a common universe U. Then we say that:

1. \((H_1, C_1)\) is a soft subset of \((H_2, C_2)\) denoted by \((H_1, C_1) \subseteq (H_2, C_2)\) if \( C_1 \subseteq C_2 \) and \( H_1(e) \subseteq H_2(e) \) for each \( e \in C_1 \).

2. The soft union of two soft sets \((H_1, C_1)\) and \((H_2, C_2)\) over a common universe U is the soft set \((H, C)\), where \( C = C_1 \cup C_2 \), and \( \forall e \in C \),

\[
H(e) = \begin{cases} 
H_1(e) & \text{if } e \in C_1 - C_2 \\
H_2(e) & \text{if } e \in C_2 - C_1 \\
H_1(e) \cup H_2(e) & \text{if } e \in C_1 \cap C_2 
\end{cases}
\]

And we write \((H, C) = (H_1, C_1) \bigcup (H_2, C_2)\).
(3) The soft intersection of two soft sets \((H_1, C_1)\) and \((H_2, C_2)\) over a common universe \(U\) is the soft set \((H, C)\), where \(C = C_1 \cap C_2\), and \(\forall e \in C, H(e) = H_1(e) \cap H_2(e)\), and we write 
\((H, C) = (H_1, C_1) \cap (H_2, C_2)\).

(4) The soft difference of two soft sets \((H_1, C_1)\) and \((H_2, C_2)\) over a common universe \(U\) is the soft set \((H, C)\), where \(C = C_1 \cap C_2\), and \(\forall e \in C, H(e) = H_1(e) - H_2(e)\), and we write 
\((H, C) = (H_1, C_1) - (H_2, C_2)\).

**Definition (1.8)** \[\text{[2]}\]: A soft topology on \(U\) is a collection \(\tilde{T}\) of soft subsets of \(\tilde{U}\) having the following properties:

(i) \(\tilde{O} \subseteq \tilde{T}\) and \(\tilde{U} \subseteq \tilde{T}\).

(ii) If \((H_1, P), (H_2, P) \in \tilde{T}\), then \((H_1, P) \cap (H_2, P) \in \tilde{T}\).

(iii) If \((H_j, P) \in \tilde{T}, \forall j \in \Omega\), then \(\bigcup_{j \in \Omega} (H_j, P) \in \tilde{T}\).

The triple \((U, \tilde{T}, P)\) is called a soft topological space over \(U\). The members of \(\tilde{T}\) are called soft open sets over \(U\). The complement of a soft open set is called soft closed.

**Definition (1.9)** \[\text{[3]}\]: Let \(U\) be a non-empty set and let \(\tilde{T}_1\) and \(\tilde{T}_2\) be two soft topologies over \(U\). Then \((U, \tilde{T}_1, \tilde{T}_2, P)\) is called a soft bitopological space over \(U\).

**Definition (1.10)** \[\text{[3]}\]: A soft subset \((H, P)\) of a soft bitopological space \((U, \tilde{T}_1, \tilde{T}_2, P)\) is called soft \(\tilde{T}_1\tilde{T}_2\)-open if \((H, P) = (H_1, P) \tilde{T} (H_2, P)\) such that \((H_1, P) \in \tilde{T}_1\) and \((H_2, P) \in \tilde{T}_2\). The complement of a soft \(\tilde{T}_1\tilde{T}_2\)-open set in \(\tilde{U}\) is called soft \(\tilde{T}_1\tilde{T}_2\)-closed.

**Definitions (1.11)** \[\text{[7]}\]: A soft bitopological space \((U, \tilde{T}_1, \tilde{T}_2, P)\) is called a soft \((1,2)^*\)-\(\tilde{T}_0\)-space if for any two distinct soft points \(\tilde{x}\) and \(\tilde{y}\) of \(\tilde{U}\), there exists a soft \(\tilde{T}_1\tilde{T}_2\)-open set in \(\tilde{U}\) containing one of the soft points but not the other.

**Definition (1.12)** \[\text{[7]}\]: A soft bitopological space \((U, \tilde{T}_1, \tilde{T}_2, P)\) is called a soft \((1,2)^*\)-\(\tilde{T}_1\)-space if every soft singleton set in \(\tilde{U}\) is either soft \(\tilde{T}_1\tilde{T}_2\)-open or soft \(\tilde{T}_1\tilde{T}_2\)-closed.

**Definition (1.13)** \[\text{[7]}\]: A soft bitopological space \((U, \tilde{T}_1, \tilde{T}_2, P)\) is called a soft \((1,2)^*\)-\(\tilde{T}_1\)-space if for any two distinct soft points \(\tilde{x}\) and \(\tilde{y}\) of \(\tilde{U}\), there exists a soft \(\tilde{T}_1\tilde{T}_2\)-open set in \(\tilde{U}\) containing \(\tilde{x}\) but not \(\tilde{y}\) and a soft \(\tilde{T}_1\tilde{T}_2\)-open set in \(\tilde{U}\) containing \(\tilde{y}\) but not \(\tilde{x}\).

**Definition (1.14)** \[\text{[7]}\]: A soft bitopological space \((U, \tilde{T}_1, \tilde{T}_2, P)\) is called a soft \((1,2)^*\)-\(\tilde{T}_2\)-space if for any two distinct soft points \(\tilde{x}\) and \(\tilde{y}\) of \(\tilde{U}\), there are two soft \(\tilde{T}_1\tilde{T}_2\)-open sets \((H, P)\) and \((K, P)\) in \(\tilde{U}\) such that \(\exists \tilde{e} \in (H, P), \tilde{y} \in (K, P)\) and \((H, P) \tilde{T} (K, P) = \tilde{O}\).

**Definition (1.15)** \[\text{[4]}\]: A soft subset \((H, P)\) of a soft bitopological space \((U, \tilde{T}_1, \tilde{T}_2, P)\) is called soft \((1,2)^*\)-omega open (briefly soft \((1,2)^*\)-\(\omega\)-open) if for each \(\tilde{x} \in (H, P)\), there exists a soft \(\tilde{T}_1\tilde{T}_2\)-open set \((O, P)\) in \(\tilde{U}\) such that \(\exists \tilde{e} \in (O, P)\) and \((O, P) \tilde{T} (H, P) = \tilde{O}\) is a countable soft set. The complement of a soft \((1,2)^*\)-\(\omega\)-open set is called soft \((1,2)^*\)-\(\omega\)-closed. Clearly, every soft \(\tilde{T}_1\tilde{T}_2\)-open set is soft \((1,2)^*\)-\(\omega\)-open, but the converse in general is not true we can see in the following example:

**Example (1.16)**: Let \(U = \{1,2,3\}\) and \(P = \{p_1, p_2\}\), and let \(\tilde{T}_1 = \{\tilde{U}, \tilde{O}, (H_1, P)\}\) and \(\tilde{T}_2 = \{\tilde{U}, \tilde{O}, (H_2, P)\}\) be soft topologies over \(U\), where \((H_1, P) = \{(p_1, \{U\}), (p_2, \{1,2\})\}\) and \((H_2, P) = \{(p_1, \{U\}), (p_2, \{1,2\})\}\).
\{(p_1,\{U\}),\{p_2,\{I\}\}\}. The soft sets in \{\tilde{U},\tilde{P},(H_1,P),(H_2,P)\} are soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open sets in \(\tilde{U}\). Thus \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is a soft bitopological space and \((H,P) = \{(p_1,\{U\}),\{p_2,\{I\}\}\}\) is a soft \((1,2)*-\omega\)-open set in \(\tilde{U}\), but is not soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open.

**Definition (1.17)** [4]: Let \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) be a soft bitopological space and \((H,P) \subseteq \tilde{U}\). Then:

(i) The soft \((1,2)*\)-omega closure (briefly soft \((1,2)*\)-omega closure) of \((H,P)\), denoted by \((1,2)*\-\omega cl(H,P)\) is the intersection of all soft \((1,2)*\)-omega closed sets in \(\tilde{U}\) which contains \((H,P)\).

(ii) The soft \((1,2)*\)-omega interior (briefly soft \((1,2)*\)-omega interior) of \((H,P)\), denoted by \((1,2)*\-\omega int(H,P)\) is the union of all soft \((1,2)*\)-omega open sets in \(\tilde{U}\) which are contained in \((H,P)\).

**Theorem (1.18)** [4]: If \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is a soft bitopological space, and \((H,P),(K,P) \subseteq \tilde{U}\). Then:

(i) \((H,P) \subseteq (1,2)*\-\omega cl(H,P) \subseteq \tilde{\tau}_1\tilde{\tau}_2 cl(H,P)\).

(ii) \((1,2)*\-\omega cl(H,P)\) is soft \((1,2)*\-omega closed in \(\tilde{U}\).

(iii) \((H,P)\) is soft \((1,2)*\-omega closed if and only if \((1,2)*\-\omega cl(H,P) = (H,P)\).

(iv) If \((H,P) \subseteq (K,P)\), then \((1,2)*\-\omega cl(H,P) \subseteq (1,2)*\-\omega cl(K,P)\).

**Definitions (1.19)** [4]: A soft subset \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) of a soft bitopological space \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is called:

(i) Soft \((1,2)*-\alpha\)-omega open if \((H,P) \subseteq (1,2)*\-\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl((1,2)*\-\omega int(H,P)))\).

(ii) Soft \((1,2)*\)-omega pre-open if \((H,P) \subseteq (1,2)*\-\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl(H,P)))\).

(iii) Soft \((1,2)*-\beta\)-omega open if \((H,P) \subseteq \tilde{\tau}_1\tilde{\tau}_2 cl((1,2)*\-\omega int(H,P)))\).

(iv) Soft \((1,2)*\)-omega bito-open if \((H,P) \subseteq \tilde{\tau}_1\tilde{\tau}_2 cl((1,2)*\-\omega int(H,P)))\).

**Proposition (1.20)** [4]: If \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is a soft bitopological space, then the following hold:

(i) Every soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open set is soft \((1,2)*\-omega-open.

(ii) Every soft \((1,2)*\-omega-open set is soft \((1,2)*\-omega-open.

(iii) Every soft \((1,2)*\-omega-open set is soft \((1,2)*\)-omega-open.

(iv) Every soft \((1,2)*\)-omega-open set is soft \((1,2)*\)-omega-open.

(v) Every soft \((1,2)*\)-omega-open set is soft \((1,2)*\)-omega-open.

**Definition (1.21)** [4]: Let \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) be a soft bitopological space and \((H,P) \subseteq \tilde{U}\). Then the soft \((1,2)*\-omega closure (resp. soft \((1,2)*\)-pre-open closure, soft \((1,2)*\)-omega closure, soft \((1,2)*\)-omega closure) of \((H,P)\), denoted by \((1,2)*\-\omega cl(H,P)\) (resp. \((1,2)*\)-pre-open cl(H,P), \((1,2)*\)-omega cl(H,P), \((1,2)*\)-omega cl(H,P)) is the intersection of all soft \((1,2)*\)-omega closed (resp. soft \((1,2)*\)-pre-open closed, soft \((1,2)*\)-omega closed) sets in \(\tilde{U}\) which contains \((H,P)\).

**Definition (1.22)** [4]: A soft subset \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) of a soft bitopological space \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is called a soft \((1,2)*\)-omega-neighbourhood (resp. soft \((1,2)*\)-omega-neighbourhood, soft \((1,2)*\)-omega-neighbourhood, soft \((1,2)*\)-omega-neighbourhood) of a soft point \(\tilde{x}\) in \(\tilde{U}\) if there exists a soft \((1,2)*\)-omega-open (resp. soft \((1,2)*\)-omega-open, soft \((1,2)*\)-omega-open, soft \((1,2)*\)-omega-open) set \((H,P)\) in \(\tilde{U}\) such that \(\tilde{x} \in (H,P) \subseteq (N,P)\).

**Definition (1.23)**[8]: Let \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) be a soft bitopological space over \(U\) and \(\varphi \neq \emptyset \subseteq U\). Then \(\tilde{\tau}_1\varphi = \{M,P \mid \tilde{T} \subseteq (M,P) \subseteq \tilde{\tau}_1\} \) and \(\tilde{\tau}_2\varphi = \{(N,P) \mid \tilde{Y} \subseteq (N,P) \subseteq \tilde{\tau}_2\}\) are called the relative soft topologies on \(\tilde{T}\) and \((Y,\tilde{\tau}_1\varphi,\tilde{\tau}_2\varphi,P)\) is called the relative soft bitopological space of \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\).

2. Soft \((1,2)*\)-Omega Separation Axioms and Weak Soft \((1,2)*\)-Omega Separation Axioms

Now, we introduce and study new types of soft separation axioms in soft bitopological spaces called soft \((1,2)*\)-omega-separation axioms and weak soft \((1,2)*\)-omega-separation axioms such as soft \((1,2)*\-\omega\)-\(\tilde{T}\)-spaces, soft \((1,2)*\-\alpha\-\omega\)-\(\tilde{T}\)-spaces, soft \((1,2)*\-\pre\-\omega\)-\(\tilde{T}\)-spaces, soft

https://doi.org/10.30526/31.2.1953  Mathematics | 140
(1,2)*-b-ω-\(\tilde{T}_i\)-spaces, and soft (1,2)*-β-ω-\(\tilde{T}_i\)-spaces, for \(i = 0, \frac{1}{2}, 1, 2\). The fundamental properties and equivalent definitions of these types of soft separation axioms also, have been studied.

**Definitions (2.1):** A soft bitopological space \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is called a soft (1,2)*-ω-\(\tilde{T}_0\)-space (resp. soft (1,2)*-α-ω-\(\tilde{T}_0\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_0\)-space, soft (1,2)*-b-ω-\(\tilde{T}_0\)-space, soft (1,2)*-β-ω-\(\tilde{T}_0\)-space) if for any two distinct soft points \(\tilde{x}\) and \(\tilde{y}\) of \(\tilde{U}\), there exists a soft (1,2)*-ω-open (resp. soft (1,2)*-α-open, soft (1,2)*-pre-open, soft (1,2)*-b-open, soft (1,2)*-β-open) set in \(\tilde{U}\) containing one of the soft points but not the other.

**Proposition (2.2):** Every soft (1,2)*-\(\tilde{T}_0\)-space is a soft (1,2)*-ω-\(\tilde{T}_0\)-space (resp. soft (1,2)*-α-\(\tilde{T}_0\)-space, soft (1,2)*-pre-\(\tilde{T}_0\)-space, soft (1,2)*-b-\(\tilde{T}_0\)-space, soft (1,2)*-β-\(\tilde{T}_0\)-space).

**Proof:** It is obvious.

**Remark (2.3):** The converse of proposition (2.2) is not true in general we can see by the following example:

**Example (2.4):** Let \(U = \{a, b, c\}\) and \(P = \{p_1, p_2\}\) and let \(\tilde{\tau}_1 = \{\tilde{U}, \tilde{\phi}, (H_1, P)\}\) be soft topologies over \(U\), where \((H_1, P) = \{(p_1, \{a\}, p_2, \{a\})\}\), \((H_2, P) = \{(p_1, \{b\}, p_2, \{b\})\}\) and \((H_3, P) = \{(p_1, \{a, b\}, p_2, \{a, b\})\}\). The soft sets in \(\{\tilde{U}, \tilde{\phi}, (H_1, P), (H_2, P), (H_3, P)\}\) are soft \(\tilde{\tau}_1\)-\(\tilde{\tau}_2\)-open. Thus \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_0\)-space (resp. soft (1,2)*-α-\(\tilde{T}_0\)-space, soft (1,2)*-pre-\(\tilde{T}_0\)-space, soft (1,2)*-b-\(\tilde{T}_0\)-space, soft (1,2)*-β-\(\tilde{T}_0\)-space), but is not soft (1,2)*-\(\tilde{T}_0\)-space, since \((p_1, \{a\}) = \tilde{x} \neq \tilde{y} = (p_2, \{a\})\), but there exists no soft \(\tilde{\tau}_1\)-\(\tilde{\tau}_2\)-open set containing one of the soft points but not the other.

**Theorem (2.9):** A soft bitopological space \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_0\)-space (resp. soft (1,2)*-α-\(\tilde{T}_0\)-space, soft (1,2)*-pre-\(\tilde{T}_0\)-space, soft (1,2)*-b-\(\tilde{T}_0\)-space, soft (1,2)*-β-\(\tilde{T}_0\)-space) if and only if (1,2)*-ocl(\(\tilde{x}\)) \(\neq\) (1,2)*-ocl(\(\tilde{y}\)) (resp. (1,2)*-α-ocl(\(\tilde{x}\)) \(\neq\) (1,2)*-ocl(\(\tilde{y}\)), (1,2)*-pre-ocl(\(\tilde{x}\)) \(\neq\) (1,2)*-ocl(\(\tilde{y}\)), (1,2)*-b-ocl(\(\tilde{x}\)) \(\neq\) (1,2)*-ocl(\(\tilde{y}\)) for any two distinct soft points \(\tilde{x}\) and \(\tilde{y}\) of \(\tilde{U}\).

**Proof:** Let \(\tilde{x}, \tilde{y} \in \tilde{U}\) such that \(\tilde{x} \neq \tilde{y}\). Since \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_0\)-space, then there exists a soft (1,2)*-ω-open set \(\tilde{\tau}_1\)-\(\tilde{\tau}_2\)-open containing \(\tilde{x}\), but not \(\tilde{y}\). Therefore \(\tilde{U} - (H,P)\) is a soft (1,2)*-ω-closed set containing \(\tilde{y}\), but not \(\tilde{x}\). Hence (1,2)*-ocl(\(\tilde{y}\)) \(\subseteq\) \(\tilde{U} - (H,P)\). Since \(\tilde{x} \notin \tilde{U} - (H,P)\), this implies that \(\tilde{x} \notin (1,2)*\)-ocl(\(\tilde{y}\)). So we get, \(\tilde{x} \notin (1,2)*\)-ocl(\(\tilde{y}\)), but \(\tilde{x} \in (1,2)*\)-ocl(\(\tilde{y}\)). Thus (1,2)*-ocl(\(\tilde{x}\)) \(\neq\) (1,2)*-ocl(\(\tilde{y}\)).

Conversely, to prove that \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_0\)-space. Let \(\tilde{x}, \tilde{y} \in \tilde{U}\) such that \(\tilde{x} \neq \tilde{y}\). Since (1,2)*-ocl(\(\tilde{x}\)) \(\neq\) (1,2)*-ocl(\(\tilde{y}\)), then there exists \(\tilde{z} \in \tilde{U}\) such that \(\tilde{z} \notin (1,2)*\)-ocl(\(\tilde{x}\)), but \(\tilde{z} \in (1,2)*\)-ocl(\(\tilde{y}\)). Suppose \(\tilde{z} \in (1,2)*\)-ocl(\(\tilde{x}\)), to show that \(\tilde{z} \notin (1,2)*\)-ocl(\(\tilde{y}\)). If \(\tilde{z} \in (1,2)*\)-ocl(\(\tilde{y}\)) \(\Rightarrow\) \(\tilde{x} \in (1,2)*\)-ocl(\(\tilde{y}\)) \(\Rightarrow\) (1,2)*-ocl(\(\tilde{x}\)) \(\subseteq\) (1,2)*-ocl(\(\tilde{y}\)) = (1,2)*-ocl(\(\tilde{y}\)). Since \(\tilde{z} \notin (1,2)*\)-ocl(\(\tilde{x}\)), \(\tilde{z} \notin (1,2)*\)-ocl(\(\tilde{y}\)) which is a contradiction. Thus \(\tilde{z} \notin (1,2)*\)-ocl(\(\tilde{y}\)) \(\Rightarrow\) \(\tilde{x} \notin \tilde{U} - (1,2)*\)-ocl(\(\tilde{y}\)), but (1,2)*-ocl(\(\tilde{y}\)) is soft (1,2)*-ω-closed, so \(\tilde{U} - (1,2)*\)-ocl(\(\tilde{y}\)) is soft (1,2)*-ω-open which contains \(\tilde{x}\), but not \(\tilde{y}\). Therefore \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_0\)-space.
Similarly, we can prove other cases.

**Theorem (2.6):** Every soft bitopological space \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)*-\omega-\tilde{T}_0\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_0\)-space).

**Proof:** Let \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) be any soft bitopological space and \(\tilde{x}, \tilde{y} \in \tilde{U}\) such that \(\tilde{x} \neq \tilde{y}\). Since \(\tilde{U} - \{\tilde{y}\}\) is a soft \((1,2)*-\omega\)-open (resp. soft \((1,2)*-\alpha-\omega\)-open, soft \((1,2)*-\omega-\tilde{T}_0\)-open, soft \((1,2)*-\beta-\omega-\tilde{T}_0\)-open) subset of \(\tilde{U}\) containing \(\tilde{x}\), but not \(\tilde{y}\). Therefore \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)*-\tilde{T}_0\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_0\)-space).

**Corollary (2.7):** Every soft subspace of a soft \((1,2)*-\omega-\tilde{T}_0\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_0\)-space) is a soft \((1,2)*-\omega-\tilde{T}_0\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_0\)-space).

**Proof:** It is obvious.

**Proposition (2.8):** If \((U, \tilde{\tau}_1, P)\) or \((U, \tilde{\tau}_2, P)\) is a soft \(\tilde{T}_0\)-space, then \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)*-\omega-\tilde{T}_0\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_0\)-space).

**Proof:** It follows from the fact \(\tilde{\tau}_i \subseteq \text{soft } \tilde{\tau}_i \tilde{\tau}_2\)-open sets in \(\tilde{U}\), \(i = 1, 2\) and proposition (2.2).

**Remark (2.9):** The converse of proposition (2.8) is not true in general in example (2.4), \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)*-\omega-\tilde{T}_0\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\omega-\tilde{T}_0\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_0\)-space), but both \((U, \tilde{\tau}_1, P)\) and \((U, \tilde{\tau}_2, P)\) are not soft \(\tilde{T}_0\)-space.

**Definition (2.10):** A soft bitopological space \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is called a soft \((1,2)*-\omega-\tilde{T}_\frac{1}{2}\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_\frac{1}{2}\)-space, soft \((1,2)*-\omega-\tilde{T}_\frac{1}{2}\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_\frac{1}{2}\)-space) if every soft singleton set in \(\tilde{U}\) is either soft \((1,2)*-\omega\)-open (resp. soft \((1,2)*-\alpha-\omega\)-open, soft \((1,2)*-\omega\)-open, soft \((1,2)*-\beta-\omega\)-open) or soft \((1,2)*-\alpha\)-open, soft \((1,2)*-\omega\)-open, soft \((1,2)*-\beta\)-open, soft \((1,2)*-\beta\)-open).

**Proposition (2.11):** Every soft \((1,2)*-\tilde{T}_\frac{1}{2}\)-space is a soft \((1,2)*-\omega-\tilde{T}_\frac{1}{2}\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_\frac{1}{2}\)-space, soft \((1,2)*-\omega-\tilde{T}_\frac{1}{2}\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_\frac{1}{2}\)-space).

**Proof:** It is obvious.

**Remark (2.12):** The converse of proposition (2.11) is not true in general. In example (2.4) \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)*-\omega-\tilde{T}_\frac{1}{2}\)-space (resp. soft \((1,2)*-\alpha-\omega-\tilde{T}_\frac{1}{2}\)-space, soft \((1,2)*-\omega-\tilde{T}_\frac{1}{2}\)-space, soft \((1,2)*-\beta-\omega-\tilde{T}_\frac{1}{2}\)-space), but is not soft \((1,2)*-\tilde{T}_\frac{1}{2}\)-space.
Proposition (2.13): Every soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space (resp. soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\beta$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space) is a soft $(1,2)^*\tilde{T}_0$-space (resp. soft $(1,2)^*\omega$-$\tilde{T}_0$-space, soft $(1,2)^*\alpha$-$\tilde{T}_0$-space, soft $(1,2)^*\beta$-$\tilde{T}_0$-space). 

Proof: Let $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ be a soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space and let $\tilde{x}, \tilde{y} \in \tilde{U}$ such that $\tilde{x} \neq \tilde{y}$. If $\{\tilde{x}\}$ is soft $(1,2)^*\alpha$-open, then $\{\tilde{x}\}$ is a soft $(1,2)^*\alpha$-open set containing $\tilde{x}$, but not $\tilde{y}$, and if $\{\tilde{x}\}$ is soft $(1,2)^*\omega$-closed, then $\tilde{U} - \{\tilde{x}\}$ is a soft $(1,2)^*\omega$-open set containing $\tilde{y}$, but not $\tilde{x}$. Therefore $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft $(1,2)^*\omega$-$\tilde{T}_0$-space. Similarly, we can prove that other cases.

Remark (2.14): The soft $(1,2)^*\omega$-$\tilde{T}_0$-space may not be soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space in general. We can see this in the following example:

Example (2.15): Let $U = \{a, b\}$ and $P = \{p_1, p_2\}$ and let $\tilde{\tau}_1 = \{\tilde{U},\tilde{\phi},(H_1, P)\}$ and $\tilde{\tau}_2 = \{\tilde{U},\tilde{\phi},(H_2, P)\}$ be soft topologies over $U$, where $(H_1, P) = \{(p_1,\{a\}),(p_2,\{b\})\}$, $(H_2, P) = \{(p_1,\{b\}),(p_2,\{b\})\}$ and $(H_3, P) = \{(p_1,\{a,b\}),(p_2,\{b\})\}$. The soft sets in $\{\tilde{U},\tilde{\phi},(H_1, P), (H_2, P), (H_3, P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets. Thus $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft $(1,2)^*\omega$-$\tilde{T}_0$-space, but is not soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space, since $\{(p_1,\{a\})\} = \{\tilde{x}\}$ is not soft $\tilde{\tau}_1\tilde{\tau}_2$-open and is not soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

Theorem (2.16): Every soft bitopological space is a soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space (resp. soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\beta$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space).

Proof: Let $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ be any soft bitopological space and $\tilde{x} \in \tilde{U}$. Since $\tilde{U} - \{\tilde{x}\}$ is a soft $(1,2)^*\alpha$-open (resp. soft $(1,2)^*\alpha$-open, soft $(1,2)^*\omega$-open, soft $(1,2)^*\beta$-open subset of $\tilde{U}$, then $\{\tilde{x}\}$ is a soft $(1,2)^*\alpha$-open (resp. soft $(1,2)^*\alpha$-open, soft $(1,2)^*\omega$-open, soft $(1,2)^*\beta$-open) subset of $\tilde{U}$. Therefore $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space (resp. soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\beta$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space).

Corollary (2.17): A soft bitopological space $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space (resp. soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\beta$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space) if it is a soft $(1,2)^*\omega$-$\tilde{T}_0$-space (resp. soft $(1,2)^*\alpha$-$\tilde{T}_0$-space, soft $(1,2)^*\beta$-$\tilde{T}_0$-space, soft $(1,2)^*\omega$-$\tilde{T}_0$-space).

Proof: It follows that from the proposition (2.13) and theorem (2.16).

Corollary (2.18): Every soft subspace of a soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space (resp. soft $(1,2)^*\alpha$-$\til{T}_{1/2}$-space, soft $(1,2)^*\beta$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\omega$-$\tilde{T}_{1/2}$-space) is a soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space (resp. soft $(1,2)^*\alpha$-$\tilde{T}_{1/2}$-space, soft $(1,2)^*\beta$-$\tilde{T}_{1/2}$-space).

Proof: It follows that from the theorem (2.16).
Proposition (2.19): If \((U, \tilde{\tau}_1, P)\) or \((U, \tilde{\tau}_2, P)\) is a soft \(\tilde{T}_{1/2}\)-space, then \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft 
\((1,2)^*-\alpha-\omega-\tilde{T}_{1/2}\)-space (resp. soft \((1,2)^*\)-pre-\(\alpha-\omega-\tilde{T}_{1/2}\)-space, soft \((1,2)^*\)-\(b-\alpha-\tilde{T}_{1/2}\)-space, soft \((1,2)^*-\beta-\omega-\tilde{T}_{1/2}\)-space).

**Proof:** It follows from the fact \(\tilde{\tau}_i \subseteq \phi\)-soft \(\tilde{\tau}_i\)-open sets in \(\tilde{U}\), \(i = 1, 2\) and proposition (2.11).

**Remark (2.20):** The converse of proposition (2.19) is not true in general in example (2.4) \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*-\alpha-\tilde{T}_{1/2}\)-space (resp. soft \((1,2)^*\)-\(\alpha-\omega-\tilde{T}_{1/2}\)-space, soft \((1,2)^*\)-pre-\(\alpha-\omega-\tilde{T}_{1/2}\)-space, soft \((1,2)^*\)-\(b-\alpha-\tilde{T}_{1/2}\)-space, soft \((1,2)^*\)-\(\beta-\omega-\tilde{T}_{1/2}\)-space), but both \((U, \tilde{\tau}_1, P)\) and 
\((U, \tilde{\tau}_2, P)\) are not soft \((1,2)^*-\tilde{T}_{1/2}\)-space.

**Definition (2.21):** A soft bitopological space \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is called a soft \((1,2)^*-\alpha-\tilde{T}_{1}\)-space (resp. soft \((1,2)^*\)-\(\alpha-\tilde{T}_{1}\)-space, soft \((1,2)^*\)-pre-\(\alpha-\tilde{T}_{1}\)-space, soft \((1,2)^*\)-\(b-\tilde{T}_{1}\)-space, soft \((1,2)^*\)-\(\beta-\tilde{T}_{1}\)-space) if for any two distinct soft points \(\tilde{x}\) and \(\tilde{y}\) of \(\tilde{U}\), there exists a soft \((1,2)^*-\alpha-\tilde{T}_{1}\)-open (resp. soft \((1,2)^*\)-\(\alpha-\tilde{T}_{1}\)-open, soft \((1,2)^*\)-pre-\(\alpha-\tilde{T}_{1}\)-open, soft \((1,2)^*\)-\(b-\tilde{T}_{1}\)-open, soft \((1,2)^*\)-\(\beta-\tilde{T}_{1}\)-open) set in \(\tilde{U}\) containing \(\tilde{x}\) but not \(\tilde{y}\) and a soft \((1,2)^*\)-\(\alpha-\tilde{T}_{1}\)-open (resp. soft \((1,2)^*\)-\(\alpha-\tilde{T}_{1}\)-open, soft \((1,2)^*\)-pre-\(\alpha-\tilde{T}_{1}\)-open, soft \((1,2)^*\)-\(b-\tilde{T}_{1}\)-open, soft \((1,2)^*\)-\(\beta-\tilde{T}_{1}\)-open) set in \(\tilde{U}\) containing \(\tilde{y}\) but not \(\tilde{x}\).

**Proposition (2.22):** Every soft \((1,2)^*-\omega-\tilde{T}_{1}\)-space (resp. soft \((1,2)^*\)-\(\omega-\tilde{T}_{1}\)-space, soft \((1,2)^*\)-\(\alpha-\omega-\tilde{T}_{1}\)-space, soft \((1,2)^*\)-pre-\(\alpha-\tilde{T}_{1}\)-space, soft \((1,2)^*\)-\(b-\tilde{T}_{1}\)-space, soft \((1,2)^*\)-\(\beta-\tilde{T}_{1}\)-space) is a soft \((1,2)^*-\omega-\tilde{T}_{1/2}\)-space (resp. soft \((1,2)^*\)-\(\omega-\tilde{T}_{1/2}\)-space, soft \((1,2)^*\)-\(\alpha-\tilde{T}_{1/2}\)-space, soft \((1,2)^*\)-pre-\(\alpha-\tilde{T}_{1/2}\)-space, soft \((1,2)^*\)-\(b-\tilde{T}_{1/2}\)-space, soft \((1,2)^*\)-\(\beta-\tilde{T}_{1/2}\)-space).

**Remark (2.23):** The soft \((1,2)^*\)-\(\tilde{T}_{1/2}\)-space may not be soft \((1,2)^*\)-\(\tilde{T}_{1}\)-space in general we can see in the following example:

**Example (2.24):** Let \(U = \{a, b\}\) and \(P = \{p\}\) and let \(\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H, P)\}\) and \(\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}, (H, P)\}\) be soft topologies over \(U\), where \((H, P) = \{(p, \{a\})\}\). The soft sets in \(\{\tilde{U}, \tilde{\varphi}, (H, P)\}\) are soft \(\tilde{\tau}_1, \tilde{\tau}_2\)-open sets. Thus \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-\(\tilde{T}_{1/2}\)-space, but is not soft \((1,2)^*\)-\(\tilde{T}_{1}\)-space, since \((p, \{a\}) = \tilde{x} \neq \tilde{y} = (p, \{b\})\), but there exists no soft \(\tilde{\tau}_1\)-open set containing \(\tilde{y}\), but not containing \(\tilde{x}\).

**Proposition (2.25):** Every soft \((1,2)^*-\tilde{T}_{1}\)-space is a soft \((1,2)^*-\alpha-\tilde{T}_{1}\)-space (resp. soft \((1,2)^*-\alpha-\omega-\tilde{T}_{1}\)-space, soft \((1,2)^*-\omega-\tilde{T}_{1}\)-space, soft \((1,2)^*-\beta-\omega-\tilde{T}_{1}\)-space, soft \((1,2)^*-\beta-\tilde{T}_{1}\)-space).

**Proof:** It is obvious.

**Remark (2.26):** The converse of proposition (2.25) is not true in general. We see that in the following example:

**Example (2.27):** Let \(U = \{a, b\}\) and \(P = \{p_1, p_2\}\) and let \(\tilde{\tau}_1 = \{\tilde{U}, \tilde{\varphi}, (H_1, P)\}\) and \(\tilde{\tau}_2 = \{\tilde{U}, \tilde{\varphi}, (H_2, P)\}\) be soft topologies over \(U\), where \((H_1, P) = \{(p_1, \{a\}), (p_2, \{b\})\}\) and \((H_2, P) = \{(p_1, \{b\}), (p_2, \{a\})\}\). The soft sets in \(\{\tilde{U}, \tilde{\varphi}, (H_1, P), (H_2, P)\}\) are soft \(\tilde{\tau}_1, \tilde{\tau}_2\)-open. Thus \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-\(\tilde{T}_{1}\)-space (resp. soft \((1,2)^*-\alpha-\tilde{T}_{1}\)-space, soft \((1,2)^*-\omega-\tilde{T}_{1}\)-space, soft \((1,2)^*-\beta-\omega-\tilde{T}_{1}\)-space), but is not soft \((1,2)^*-\tilde{T}_{1}\)-space.
Theorem (2.28): In a soft bitopological space \((U, \tau_1, \tau_2, P)\) the following statements are equivalent.

(i) \((U, \tau_1, \tau_2, P)\) is a soft \((1,2)*\)-\(\alpha\)-\(\beta\)-\(\omega\) \(\tilde{T}_1\)-space (resp. soft \((1,2)*\)-\(\alpha\)-\(\omega\) \(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\beta\)-\(\omega\) \(\tilde{T}_1\)-space)

(ii) For each \(x \in \tilde{U}\), \(\{x\}\) is a soft \((1,2)*\)-\(\alpha\)-closed (resp. soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\beta\)-\(\omega\)-\(\tilde{T}_1\)-space)

(iii) Every soft subset of \(\tilde{U}\) is the intersection of all soft \((1,2)*\)-\(\alpha\)-\(\omega\)-open, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\beta\)-\(\omega\)-\(\tilde{T}_1\)-space

(iv) The intersection of all soft \((1,2)*\)-\(\alpha\)-\(\omega\)-open, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\beta\)-\(\omega\)-\(\tilde{T}_1\)-space) sets containing the soft point \(x \in \tilde{U}\) is \(\{x\}\).

Proof: (i) \(\Rightarrow\) (ii). Let \(\tilde{x} \in \tilde{U}\). To prove that \(\{\tilde{x}\}\) is soft \((1,2)*\)-\(\alpha\)-closed in \(\tilde{U}\). Let \(\tilde{y} \in \{\tilde{x}\}\) \(\Rightarrow\) \(\tilde{x} \neq \tilde{y}\). Since \((U, \tau_1, \tau_2, P)\) is a soft \((1,2)*\)-\(\alpha\)-\(\tilde{T}_1\)-space, there is a soft \((1,2)*\)-\(\alpha\)-\(\tilde{T}_1\)-space \((H, P)\) in \(\tilde{U}\) such that \(\tilde{y} \in \{\tilde{x}\}\), but \(\tilde{y} \notin \{\tilde{x}\}\) \(\Rightarrow\) \(\{\tilde{x}\}\) \(\subseteq\) \((H, P)^c \Rightarrow (2.28) \Rightarrow (1.2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space. Similarly, we can prove that other cases.

Theorem (2.29): Every soft bitopological space is a soft \((1,2)*\)-\(\alpha\)-\(\omega\) \(\tilde{T}_1\)-space (resp. soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space).

Proof: Let \((U, \tau_1, \tau_2, P)\) be any soft bitopological space and \(\tilde{x}, \tilde{y} \in \tilde{U}\) such that \(\tilde{x} \neq \tilde{y}\). Since \(\tilde{U} - \{\tilde{x}\}\) and \(\tilde{U} - \{\tilde{y}\}\) are soft \((1,2)*\)-\(\alpha\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\beta\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space) sets in \(\tilde{U}\) such that \(\tilde{U} - \{\tilde{y}\}\) containing \(\tilde{x}\), but not \(\tilde{y}\) and \(\tilde{U} - \{\tilde{x}\}\) containing \(\tilde{y}\), but not \(\tilde{x}\). Therefore \((U, \tau_1, \tau_2, P)\) is a soft \((1,2)*\)-\(\alpha\)-\(\tilde{T}_1\)-space (resp. soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\beta\)-\(\omega\)-\(\tilde{T}_1\)-space).

Corollary (2.30): A soft bitopological space \((U, \tau_1, \tau_2, P)\) is a soft \((1,2)*\)-\(\alpha\)-\(\omega\) \(\tilde{T}_1\)-space (resp. soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space) iff it is a soft \((1,2)*\)-\(\alpha\)-\(\omega\) \(\tilde{T}_1\)-space (resp. soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\beta\)-\(\tilde{T}_1\)-space).

Proof: It follows that from the proposition (2.22) and theorem (2.29).

Corollary (2.31): Every soft subspace of a soft \((1,2)*\)-\(\alpha\)-\(\omega\) \(\tilde{T}_1\)-space (resp. soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\beta\)-\(\tilde{T}_1\)-space, soft \((1,2)*\)-\(\omega\)-\(\tilde{T}_1\)-space) is a
soft (1,2)*-ω-\(\tilde{T}_1\)-space (resp. soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_1\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_1\)-space, soft (1,2)*-b-ω-\(\tilde{T}_1\)-space, soft (1,2)*-\(\beta\)-ω-\(\tilde{T}_1\)-space).

**Proof:** It follows that from the theorem (2.29).

**Proposition (2.32):** If \((U, \tilde{\tau}_1, P)\) or \((U, \tilde{\tau}_2, P)\) is a soft \(\tilde{T}_1\)-space, then \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_1\)-space (resp. soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_1\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_1\)-space, soft (1,2)*-b-ω-\(\tilde{T}_1\)-space, soft (1,2)*-\(\beta\)-ω-\(\tilde{T}_1\)-space).

**Proof:** It follows from the fact \(\tilde{\tau}_1 \subseteq \tilde{\tau}_i \tilde{\tau}_2\)-open sets in \(\tilde{U}\), \(i = 1,2\) and proposition (2.25).

**Remark (2.33):** The converse of proposition (2.32) is not true in general in example (2.27) \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_1\)-space (resp. soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_1\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_1\)-space, soft (1,2)*-b-ω-\(\tilde{T}_1\)-space, soft (1,2)*-\(\beta\)-ω-\(\tilde{T}_1\)-space), but both \((U, \tilde{\tau}_1, P)\) and \((U, \tilde{\tau}_2, P)\) are not soft \(\tilde{T}_1\)-space.

**Definition (2.34):** A soft bitopological space \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is called a soft (1,2)*-ω-\(\tilde{T}_2\)-space (resp. soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_2\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_2\)-space, soft (1,2)*-b-ω-\(\tilde{T}_2\)-space, soft (1,2)*-\(\beta\)-ω-\(\tilde{T}_2\)-space) if for any two distinct soft points \(\tilde{x}\) and \(\tilde{y}\) of \(\tilde{U}\), there are two soft (1,2)*-ω-open (resp. soft (1,2)*-\(\alpha\)-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-\(\beta\)-ω-open) sets \((H,P)\) and \((K,P)\) in \(\tilde{U}\) such that \(\tilde{x} \in (H,P), \tilde{y} \in (K,P)\) and \((H,P) \cap (K,P) = \emptyset\).

**Proposition (2.35):** Every soft (1,2)*-ω-\(\tilde{T}_2\)-space (resp. soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_2\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_2\)-space, soft (1,2)*-b-ω-\(\tilde{T}_2\)-space, soft (1,2)*-\(\beta\)-ω-\(\tilde{T}_2\)-space) is a soft (1,2)*-ω-\(\tilde{T}_1\)-space (resp. soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_1\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_1\)-space, soft (1,2)*-b-ω-\(\tilde{T}_1\)-space, soft (1,2)*-\(\beta\)-ω-\(\tilde{T}_1\)-space).

**Proof:** Let \(\tilde{x}, \tilde{y} \in \tilde{U}, \tilde{x} \neq \tilde{y}\). By our assumption there are two soft (1,2)*-ω-open sets \((H,P)\) and \((K,P)\) in \(\tilde{U}\) such that \(\tilde{x} \in (H,P), \tilde{y} \in (K,P)\) and \((H,P) \cap (K,P) = \emptyset\). Thus \((H,P)\) and \((K,P)\) are soft (1,2)*-ω-open sets in \(\tilde{U}\) such that \((H,P)\) containing \(\tilde{x}\), but not \(\tilde{y}\) and \((K,P)\) containing \(\tilde{y}\), but not \(\tilde{x}\). Therefore \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_1\)-space. Similarly, we can prove that other cases.

**Remark (2.36):** The converse of proposition (2.35) is not true in general. We see that in the following example:

**Example (2.37):** Let \(X = \mathcal{R}\) and \(P = \{p_1, p_2\}\) and let \(\tilde{\tau}_1 = \{(H,P) \subseteq \tilde{U} : (H,P)^c\) is finite\} \(\tilde{U}\) \(\emptyset\) and \(\tilde{\tau}_2 = \{\tilde{U}, \emptyset\}\) be soft topologies over \(U\). Thus \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft (1,2)*-ω-\(\tilde{T}_1\)-space (resp. soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_1\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_1\)-space, soft (1,2)*-b-ω-\(\tilde{T}_1\)-space, soft (1,2)*-\(\beta\)-ω-\(\tilde{T}_1\)-space), clear that is not soft (1,2)*-b-ω-\(\tilde{T}_2\)-space.

**Proposition (2.38):** (i) Every soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_2\)-space is a soft (1,2)*-ω-\(\tilde{T}_2\)-space.

(ii) Every soft (1,2)*-ω-\(\tilde{T}_2\)-space is a soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_2\)-space.

(iii) Every soft (1,2)*-\(\alpha\)-ω-\(\tilde{T}_2\)-space is a soft (1,2)*-pre-ω-\(\tilde{T}_2\)-space.

(iv) Every soft (1,2)*-pre-ω-\(\tilde{T}_2\)-space is a soft (1,2)*-b-ω-\(\tilde{T}_2\)-space.

(v) Every soft (1,2)*-b-ω-\(\tilde{T}_2\)-space is a soft (1,2)*-\(\beta\)-ω-\(\tilde{T}_2\)-space.

**Remark (2.39):** The converse of proposition (2.38) is not true in general as shown by the following examples:
Example (2.40): Let $U = \{a, b, c\}$ and $P = \{p_1, p_2\}$ and let $\tilde{\tau}_1 = (\tilde{U}, \tilde{\phi}, (H_1, P))$ and $\tilde{\tau}_2 = (\tilde{U}, \tilde{\phi}, (H_2, P))$ be soft topologies over $U$, where $(H_1, P) = \{(p_1, \{a\}), (p_2, \{a\})\}$ and $(H_2, P) = \{(p_1, \{b\}), (p_2, \{b, c\})\}$. The soft sets in $\{\tilde{U}, \tilde{\phi}, (H_1, P), (H_2, P)\}$ are soft $\tilde{\tau}_2$-open. Thus $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft $(1,2)^{*}$-ω-$\tilde{T}_2$-space, but is not soft $(1,2)^{*}$- $\tilde{T}_2$-space. Since $(p_1, \{a\}) = \tilde{x}$ ≠ $\tilde{y} = (p_2, \{a\})$, but there exists no soft $\tilde{x}$-$\tilde{T}_2$-open set $(K_1, P)$ containing $\tilde{x}$ and soft $\tilde{\tau}_2$-open set $(K_2, P)$ containing $\tilde{y}$ such that $(K_1, P) \tilde{\cap} (K_2, P) = \tilde{\phi}$.

Example (2.41): Let $U = \mathcal{R}$ and $P = \{p\}$ and let $\tilde{\tau}_1 = (\tilde{U}, \tilde{\phi}, (H_1, P))$ and $\tilde{\tau}_2 = (\tilde{U}, \tilde{\phi}, (H_2, P))$ be soft topologies over $U$, where $(H_1, P) = \{(p, \{-1\})\}$, $(H_2, P) = \{(p, \{1\})\}$ and $(H_3, P) = \{(p, \{1, -1\})\}$. The soft sets in $\{\tilde{U}, \tilde{\phi}, (H_1, P), (H_2, P), (H_3, P)\}$ are soft $\tilde{\tau}_2$-open. Thus $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft $(1,2)^{*}$-α-$\omega$-$\tilde{T}_2$-space, clear that is not soft $(1,2)^{*}$-ω-$\tilde{T}_2$-space.

Example (2.42): Let $U = \mathcal{R}$ and $P = \{p\}$ and let $\tilde{\tau}_1 = (\tilde{U}, \tilde{\phi}, (H, P))$ and $\tilde{\tau}_2 = (\tilde{U}, \tilde{\phi}, (H, P))$ be soft topologies over $U$, where $(H, P) = \{(p, \{1\})\}$. The soft sets in $\{\tilde{U}, \tilde{\phi}, (H, P)\}$ are soft $\tilde{\tau}_2$-open. Thus $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft $(1,2)^{*}$-α-$\omega$-$\tilde{T}_2$-space, but is not soft $(1,2)^{*}$-α-$\omega$-$\tilde{T}_2$-space. Since $(p, \{2\}) = \tilde{x} \neq \tilde{y} = (p, \{3\})$, but there exists no a soft $(1,2)^{*}$-α-$\omega$-open set $(K_1, P)$ containing $\tilde{x}$ and a soft $(1,2)^{*}$-α-$\omega$-open set $(K_2, P)$ containing $\tilde{y}$ such that $(K_1, P) \tilde{\cap} (K_2, P) = \tilde{\phi}$.

Theorem (2.43): For a soft bitopological space $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ the following statements are equivalent.

(i) $(U, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft $(1,2)^{*}$-α-$\omega$-$\tilde{T}_2$-space (resp. soft $(1,2)^{*}$-α-$\omega$-$\tilde{T}_2$-space, soft $(1,2)^{*}$-α-$\omega$-$\tilde{T}_2$-space, soft $(1,2)^{*}$-α-$\omega$-$\tilde{T}_2$-space) (ii) If $\tilde{x} \in \tilde{U}$, then for each $\tilde{y} \neq \tilde{x}$, there is a soft $(1,2)^{*}$-$\omega$-neighborhood (resp. soft $(1,2)^{*}$-$\omega$-neighborhood, soft $(1,2)^{*}$-$\omega$-neighborhood, soft $(1,2)^{*}$-$\omega$-neighborhood) $(N, P)$ of $\tilde{x}$ such that $\tilde{y} \in (1,2)^{*}$-$\omega$-ocl $(N, P)$ (resp. $\tilde{y} \in (1,2)^{*}$-$\omega$-ocl $(N, P)$, $\tilde{y} \in (1,2)^{*}$-$\omega$-ocl $(N, P)$, $\tilde{y} \in (1,2)^{*}$-$\omega$-ocl $(N, P)$).

(iii) For each $\tilde{x} \in \tilde{U}$, $\tilde{\cap} \{(1,2)^{*}$-$\omega$-ocl $(N, P)$: $(N, P)$ is a soft $(1,2)^{*}$-$\omega$-neighborhood of $\tilde{x} \}$ (resp. $\tilde{\cap} \{(1,2)^{*}$-$\omega$-ocl $(N, P)$: $(N, P)$ is soft $(1,2)^{*}$-$\omega$-neighborhood of $\tilde{x} \}$, $\tilde{\cap} \{(1,2)^{*}$-$\omega$-ocl $(N, P)$: $(N, P)$ is a soft $(1,2)^{*}$-$\omega$-neighborhood of $\tilde{x} \}$, $\tilde{\cap} \{(1,2)^{*}$-$\omega$-ocl $(N, P)$: $(N, P)$ is soft $(1,2)^{*}$-$\omega$-neighborhood of $\tilde{x} \}$) = $\{\tilde{x}\}$.

Proof: (i) $\Rightarrow$ (ii). Let $\tilde{x} \in \tilde{U}$. If $\tilde{y} \in \tilde{U}$ such that $\tilde{y} \neq \tilde{x}$, then there exists disjoint soft $(1,2)^{*}$-$\omega$-open sets $(H, P)$ and $(K, P)$ such that $\tilde{x} \in (H, P)$ and $\tilde{y} \in (K, P)$. Hence $\tilde{x} \subseteq (H, P) \subseteq (K, P)$ which implies that $(K, P)$ is a soft $(1,2)^{*}$-$\omega$-neighborhood of $\tilde{x}$. Also $(K, P)$ is soft $(1,2)^{*}$-$\omega$-closed and $\tilde{y} \in (K, P)$. Let $(N, P) = (K, P)$. Then $\tilde{y} \in (1,2)^{*}$-$\omega$-ocl $(N, P)$.

(ii) $\Rightarrow$ (iii). Obvious.

(iii) $\Rightarrow$ (i). Let $\tilde{x}, \tilde{y} \in \tilde{U}$, $\tilde{x} \neq \tilde{y}$. By hypothesis, there is at least a soft $(1,2)^{*}$-$\omega$-neighborhood $(N, P)$ of $\tilde{x}$ such that $\tilde{y} \in (1,2)^{*}$-$\omega$-ocl $(N, P)$. We have $\tilde{x} \in (1,2)^{*}$-$\omega$-ocl $(N, P)^c$ which is soft $(1,2)^{*}$-$\omega$-open. Since $(N, P)$ is a soft $(1,2)^{*}$-$\omega$-neighborhood of $\tilde{x}$, there exists a soft $(1,2)^{*}$-$\omega$-open set $(H, P)$ in $\tilde{U}$ such that $\tilde{x} \subseteq (H, P) \subseteq (N, P)$ and $(H, P) \tilde{\cap} ((1,2)^{*}$-$\omega$-ocl $(N, P))^c$ = $\tilde{\phi}$. Hence $(U, \tilde{T}_1, \tilde{T}_2, P)$ is a soft $(1,2)^{*}$-$\omega$-$\tilde{T}_2$-space. Similarly, we can prove that other cases.

https://doi.org/10.30526/31.2.1953
Now, we need the following lemma.

**Lemma (2.44):** Let \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) be a soft bitopological space and \(Y \subseteq U\). If \((H, P)\) is a soft \((1,2)^*\)-\(\omega\)-open set in \(U\), then \((H, P) \cap Y\) is a soft \((1,2)^*\)-\(\omega\)-open set in \(Y\).

**Proof:** Let \((H, P)\) be a soft \((1,2)^*\)-\(\omega\)-open set in \(U\). To prove that \((H, P) \cap Y\) is a soft \((1,2)^*\)-\(\omega\)-open set in \(Y\). Let \(\tilde{Y} \subseteq (H, P) \cap Y\). Since \((H, P)\) is soft \((1,2)^*\)-\(\omega\)-open in \(U\) \(\Rightarrow \exists\) a soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open set \((V, P)\) in \(U\) such that \(\tilde{Y} \subseteq (V, P)\) and \((V, P) \cap (H, P)\) is a soft countable set. Hence \(\tilde{Y} \cap (V, P)\) is a soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open set in \(Y\). Since \(\tilde{Y} \cap ((V, P) \cap (H, P)) \subseteq ((V, P) \cap (H, P))\), then \(\tilde{Y} \cap ((V, P) \cap (H, P))\) is soft countable. Thus \((H, P)\) is a soft \((1,2)^*\)-\(\omega\)-open set in \(Y\).

**Proposition (2.45):** Every soft subspace of a soft \((1,2)^*\)-\(\omega\)-\(\tilde{T}_2\)-space is a soft \((1,2)^*\)-\(\omega\)-\(\tilde{T}_2\)-space.

**Proof:** Let \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) be a soft \((1,2)^*\)-\(\omega\)-\(\tilde{T}_2\)-space and \((Y, \tilde{\tau}_1Y, \tilde{\tau}_2Y, P)\) be a soft subspace of \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\). To prove that \((Y, \tilde{\tau}_1Y, \tilde{\tau}_2Y, P)\) is a soft \((1,2)^*\)-\(\omega\)-\(\tilde{T}_2\)-space. Let \(\tilde{x}, \tilde{y} \in \tilde{Y}\) such that \(\tilde{x} \neq \tilde{y}\). Since \(\tilde{Y} \subseteq U\), then \(\tilde{x}, \tilde{y} \in U\). But \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-\(\omega\)-\(\tilde{T}_2\)-space, then there are two soft \((1,2)^*\)-\(\omega\)-open sets \((H, P)\) and \((K, P)\) in \(U\) such that \(\tilde{x} \in (H, P)\) and \(\tilde{y} \in (K, P)\). By lemma (2.44), \((H', P) = (H, P) \cap \tilde{Y}\) and \((K', P) = (K, P) \cap \tilde{Y}\) are soft \((1,2)^*\)-\(\omega\)-open sets in \(Y\) such that \(\tilde{x} \in (H', P)\) and \(\tilde{y} \in (K', P)\). Since \((H', P) \cap (K', P) = \tilde{Y}\) \(\Rightarrow \tilde{Y} \subseteq (H, P) \cap (K, P)\), \((H, P) \cap (K, P) = \tilde{Y}\) \(\Rightarrow \tilde{Y} \subseteq (H', P) \cap (K', P)\). Thus \((Y, \tilde{\tau}_1Y, \tilde{\tau}_2Y, P)\) is a soft \((1,2)^*\)-\(\omega\)-\(\tilde{T}_2\)-space.

**Remark (2.46):** Soft subspace of a soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_2\)-space is not a soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-\(\til{T}_2\)-space as shown in the following example:

**Example (2.47):** Let \(U = \mathfrak{R}\) and \(P = \{p\}\) and let \(\tilde{\tau}_1 = \{\tilde{U}, \tilde{\phi}, (H_1, P)\}\) and \(\tilde{\tau}_2 = \{\tilde{U}, \tilde{\phi}, (H_2, P)\}\) be soft topologies over \(U\), where \((H_1, P) = \{(p, [-1])\}\), \((H_2, P) = \{(p, [1])\}\) and \((H_3, P) = \{(p, [1, -1])\}\). Then \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_2\)-space. If \(Y = \mathfrak{R} - \{1\} \subseteq U = \mathfrak{R}\), then \(\tilde{\tau}_1Y = \{\tilde{Y}, \tilde{\phi}, (H_1, P)\}\) and \(\tilde{\tau}_2Y = \{\tilde{Y}, \tilde{\phi}\}\) are soft topologies over \(Y\). The soft sets in \(\tilde{Y}\) \(\subseteq Y\) are soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open. Therefore \((Y, \tilde{\tau}_1Y, \tilde{\tau}_2Y, P)\) is not a soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_2\)-space, since \((p, [3]) = \tilde{x} \neq \tilde{y} = (p, [4])\), but there exists no soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open sets \((K_1, P)\) and \((K_2, P)\) in \(\tilde{Y}\) such that \(\tilde{x} \in (K_1, P)\) and \(\tilde{y} \in (K_2, P)\) and \((K_1, P) \cap (K_2, P) = \emptyset\).

**Proposition (2.48):** If \((U, \tilde{\tau}_1, P)\) or \((U, \tilde{\tau}_2, P)\) is a soft \(\tilde{T}_2\)-space, then \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-\(\omega\)-\(\tilde{T}_2\)-space (resp. soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_2\)-space, soft \((1,2)^*\)-pre-\(\omega\)-\(\tilde{T}_2\)-space, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-\(\tilde{T}_2\)-space, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-\(\tilde{T}_2\)-space).

**Proof:** It follows from the fact \(\tilde{\tau}_1 \subseteq\) soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open sets in \(U\), \(i = 1, 2\) and proposition (2.38).

**Remark (2.49):** The converse of proposition (2.48) is not true in general. We see that by the following example:

**Example (2.50):** Let \(U = \{a, b, c, d\}\) and \(P = \{p_1, p_2\}\) and let \(\tilde{\tau}_1 = \{\tilde{U}, \tilde{\phi}, (H_1, P)\}\) and \(\tilde{\tau}_2 = \{\tilde{U}, \tilde{\phi}, (H_2, P)\}\) be soft topologies over \(U\), where \((H_1, P) = \{(p_1, [a, b]), (p_2, [a, b])\}\) and \((H_2, P) = \{(p_1, [a]), (p_2, [a])\}\). The soft sets in \(\tilde{U}\) \(\subseteq U\) are soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open. Thus \((U, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-\(\omega\)-\(\tilde{T}_2\)-space (resp. soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-\(\tilde{T}_2\)-space, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-\(\tilde{T}_2\)-space, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-\(\tilde{T}_2\)-space).

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pre-ω-\(\tilde{T}_2\)-space, soft (1,2)*-b-ω-\(\tilde{T}_2\)-space, soft (1,2)*-β-ω-\(\tilde{T}_2\)-space), but both (U,\(\tilde{\tau}_1\),P) and (U,\(\tilde{\tau}_2\),P) are not soft \(\tilde{T}_2\)-space.

**Definition (2.51):** A soft function \(f : (U,\tilde{\tau}_1,\tilde{\tau}_2,P) \rightarrow (V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) is called strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) if \(f^{-1}((H,P))\) is a soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open set in \(\tilde{U}\) for each soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set \((H,P)\) in \(\tilde{V}\).

**Theorem (2.52):** Let \(f : (U,\tilde{\tau}_1,\tilde{\tau}_2,P) \rightarrow (V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) be a strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) injective function. If \((V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) is a soft (1,2)*-ω-\(\tilde{T}_1\)-space (resp. soft (1,2)*-α-ω-\(\tilde{T}_1\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_1\)-space, soft (1,2)*-b-ω-\(\tilde{T}_1\)-space, soft (1,2)*-β-ω-\(\tilde{T}_1\)-space), then \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is a soft (1,2)*-\(\tilde{T}_1\)-space, for \(i = 0,\frac{1}{2},1,2\).

**Proof:** Suppose that \((V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) is a soft (1,2)*-ω-\(\tilde{T}_2\)-space. Let \(\tilde{x},\tilde{y} \in \tilde{U}\) such that \(\tilde{x} \neq \tilde{y}\). Since \(f\) is injective and \((V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) is a soft (1,2)*-ω-\(\tilde{T}_2\)-space, then there exists disjoint soft (1,2)*-ω-open sets \((H_1,P)\) and \((H_2,P)\) in \(\tilde{V}\) such that \(f(\tilde{x}) \notin (H_1,P)\) and \(f(\tilde{y}) \in (H_2,P)\).

By definition (2.51), \(f^{-1}((H_1,P))\) and \(f^{-1}((H_2,P))\) are soft \(\tilde{\tau}_1\tilde{\tau}_2\)-open sets in \(\tilde{U}\) such that \(\tilde{x} \in f^{-1}((H_1,P)), \tilde{y} \notin f^{-1}((H_2,P))\) and \(f^{-1}((H_1,P))\) is called strongly soft (1,2)*-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set \((H,P)\) in \(\tilde{U}\).

**Definition (2.53):** A soft function \(f : (U,\tilde{\tau}_1,\tilde{\tau}_2,P) \rightarrow (V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) is called strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) if \(f((H,P))\) is a soft \(\tilde{\sigma}_1\tilde{\sigma}_2\)-open set in \(\tilde{V}\) for each soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set \((H,P)\) in \(\tilde{U}\).

**Theorem (2.54):** Let \(f : (U,\tilde{\tau}_1,\tilde{\tau}_2,P) \rightarrow (V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) be a strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) bijective function. If \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is a soft (1,2)*-ω-\(\tilde{T}_1\)-space (resp. soft (1,2)*-α-ω-\(\tilde{T}_1\)-space, soft (1,2)*-pre-ω-\(\tilde{T}_1\)-space, soft (1,2)*-b-ω-\(\tilde{T}_1\)-space, soft (1,2)*-β-ω-\(\tilde{T}_1\)-space), then \((V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) is a soft (1,2)*-\(\tilde{T}_1\)-space, for \(i = 0,\frac{1}{2},1,2\).

**Proof:** Suppose that \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is a soft (1,2)*-ω-\(\tilde{T}_2\)-space. Let \(\tilde{y}_1,\tilde{y}_2 \in \tilde{V}\) such that \(\tilde{y}_1 \neq \tilde{y}_2\). Since \(f\) is surjective, then there exists \(\tilde{x}_1,\tilde{x}_2 \in \tilde{U}\) such that \(f(\tilde{x}_1) = \tilde{y}_1\) and \(f(\tilde{x}_2) = \tilde{y}_2\). Since \(f\) is a function, then \(\tilde{x}_1 \neq \tilde{x}_2\). But \((U,\tilde{\tau}_1,\tilde{\tau}_2,P)\) is a soft (1,2)*-ω-\(\tilde{T}_2\)-space, then there exists disjoint soft (1,2)*-ω-open sets \((H_1,P)\) and \((H_2,P)\) in \(\tilde{U}\) such that \(\tilde{x}_1 \notin (H_1,P)\) and \(\tilde{x}_2 \notin (H_2,P)\). By definition (2.53), \(f((H_1,P))\) and \(f((H_2,P))\) are soft \(\tilde{\sigma}_1\tilde{\sigma}_2\)-open sets in \(\tilde{V}\) such that \(f(\tilde{x}_1) \notin (H_1,P)\) and \(f(\tilde{x}_2) \notin (H_2,P)\). Since \(f\) is injective, then \(f((H_1,P)) = f((H_2,P)) = \emptyset\). Hence \((V,\tilde{\sigma}_1,\tilde{\sigma}_2,P)\) is a soft (1,2)*-\(\tilde{T}_2\)-space. By the same way we can prove that other cases.

The following diagram shows the relation among soft (1,2)*-\(\tilde{T}_1\)-spaces, soft (1,2)*-ω-\(\tilde{T}_1\)-spaces, soft (1,2)*-α-ω-\(\tilde{T}_1\)-spaces, soft (1,2)*-pre-ω-\(\tilde{T}_1\)-spaces, soft (1,2)*-b-ω-\(\tilde{T}_1\)-spaces, and soft (1,2)*-β-ω-\(\tilde{T}_1\)-spaces, for \(i = 0,\frac{1}{2},1,2\).

https://doi.org/10.30526/31.2.1953
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https://doi.org/10.30526/31.2.1953